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FORMATION OF A THERMAL STRUCTURE IN AN INHOMOGENEOUS METAL CONDUCTOR UNDER A HIGH-DENSITY ELECTRIC CURRENT

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The heating of a metal conductor with a localized inhomogeneous inclusion by a high-density electric current is discussed.

A considerable number of papers on thermal phenomena in nonlinear media have recently been published (e.g., [1-4]). So-called thermal structures, i.e., localized spatially nonuniform steady-state temperature distributions, have been found to form under certain conditions in the regions of space where the processes are fastest. The structures can form when initially the nonlinear medium is spatially uniform and the temperature distribution is nonuniform [1, 2] as well as in the opposite case [3, 4]. The mathematical differences are evidenced by the fact of the dependence of the heat equation explicitly only on the temperature in the first case but also on the spatial variables in the second case. To my knowledge, no one has yet tackled problems of the second type with an analytical approach.

This paper considers the heating of a metal conductor, containing an inhomogeneous inclusion (inhomogeneity), by a high-density electric current. Inhomogeneity is construed as a region of space where the physical characteristics (in the given case, the electrical conductivity of the conductor) differ from those of the ambient medium.

A similar problem was solved in [3, 4] by computer simulation of the medium using a network of nonlinear resistors with randomly distributed inhomogeneities. It was shown that with time these inhomogeneities extend (intergrow) across the current lines. The problem, however, was solved on the assumption of zero thermal conductivity of the medium. The distributions of the electric current and the temperature in a conducting medium with inhomogeneities in the initial stage were considered in [5].

Below we make a qualitative analysis of the dynamics of the intergrowth of a single inhomogeneity in a heat-conducting medium with Joule dissipation of energy, when the electrical conductivity of the inhomogeneity differs little from that of the medium. The mathematical problem is formulated as follows.

An electric current of density **j** flows through an infinite metal medium (conductor) with an inhomogeneity located at

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the origin and having a geometric shape described by the function $\Omega(r, z)$. The current is directed along the z axis at an infinite distance from the inhomogeneity. We consider conductors of simple metals, for which the temperature dependence of the electrical conductivity has the form [6]

$$\sigma = [1 + a\Omega(r, z)]^{-1}T^{-1}. \tag{1}$$

Here σ is the dimensionless electrical conductivity at the point (r, z) relative to σ_0, the initial electrical conductivity of the conductor at infinity, $T = 1 + \alpha(T_m - T_0)$; T_m and T_0 are the instantaneous and initial temperature of the medium, $a = (\sigma_0/\sigma - 1)$ at $T = 1$, and $r = z = 0$ is the inhomogeneity parameter. The other physical quantities, i.e., the thermal diffusivity χ, density γ, and heat capacity c, are assumed to be constant. It is assumed that no heat transfer occurs between the conductor and the ambient medium.

The system of equations describing the distribution of the temperature and the electric field will include the electric field equation curl $\mathbf{E} = 0$, div $\mathbf{j} = 0$, $\mathbf{j} = \sigma\mathbf{E}$, $\mathbf{E} = -$ grad Φ, the heat equation

$$\frac{\partial T}{\partial t} = \nabla^2 T + \delta_0 j^2/\sigma$$

and Eq. (1). The equations are written in dimensionless form: r, z are the coordinates expressed in units of the characteristic inhomogeneity dimension R; $\mathbf{j}$, $\mathbf{E}$, Φ are current density, electric field strength, and electrical potential at the point (r, z) expressed in units of J_0 (the amplitude of the current density), J_0/σ_0, and RJ_0/σ_0, respectively; and t is the time expressed in R^2/χ. The parameter $\delta_0 = R^2\alpha J_0^2/(\chi\gamma c\sigma_0)$ is the ratio of the characteristic rates of Joule heating and heat removal due to thermal conductivity.

The system of equations can be written as

$$\nabla^2\Phi = -(\mathrm{grad}\,\Phi\,\mathrm{grad}\,\sigma)/\sigma, \tag{2}$$

$$\frac{\partial T}{\partial t} = \nabla^2 T + \delta_0\sigma\,\mathrm{grad}\,\Phi\,\mathrm{grad}\,\Phi, \tag{3}$$

$$\sigma = [1 + a\Omega(r, z)]^{-1}T^{-1} \tag{4}$$

with the initial condition

$$T(r, z, t = 0) = 1. \tag{5}$$

The solution of the problem (2)-(5) will be sought, with the condition that the inhomogeneity parameter a is small, as an asymptotic expansion

$$\Phi = j_0(t)\exp\left(\delta_0 \int j_0^2(t)\,dt\right)(-z + a\Phi_1 + O(a^2)), \tag{6}$$

$$T = \exp\left(\delta_0 \int j_0^2(t)\,dt\right)(1 + aT_1 + O(a^2)), \tag{7}$$

$j_0(t)$ is the dimensionless current density at infinity. Substituting (6) and (7) into (2)-(5) and disregarding terms of the order of a^2, we obtain a system of equations in Φ_1 and T_1,

$$\nabla^2\Phi_1 = -\frac{\partial T_1}{\partial z} - \frac{\partial\Omega}{\partial z}, \tag{8}$$

$$\frac{\partial T_1}{\partial t} = \nabla^2 T_1 - \delta_0 j_0^2(t)\left(2\frac{\partial\Phi_1}{\partial z} + 2T_1 + \Omega\right) \tag{9}$$

at zero initial condition, zero conditions at infinity, and finite values of Φ_1 and T_1 at the center. For definiteness we assume that $\Omega(r, z) = \exp(-r^2 - z^2)$. By virtue of the axial symmetry in cylindrical coordinates the solution of the system (8), (9) depends only on the two coordinates r and z. Using the Fourier and Bessel integral transforms (in z and r, respectively), we can easily obtain the solution of (8), (9), which at the center $r = z = 0$ has the form

$$T_1(t) = \frac{\delta_0}{2\sqrt{\pi}} \int_0^{\pi/2} \cos 2\theta \sin\theta d\theta \int_0^{\infty} \rho^2 \exp\left(-\frac{\rho^2}{4} - \rho^2 t - h(t)\sin^2\theta\right) J(t)\, d\rho$$

and a similar one for Φ_1, where

$$h(t) = 2\delta_0 \int_0^t j_0^2(t)\, dt, \quad J(t) = \int_0^t j_0^2(y) \exp(\rho^2 y + h(y)\sin^2\theta)\, dy.$$

As $t \to \infty$ we have $J(t) \to j_0^2(t)\exp(\rho^2 t + h(t)\sin^2\theta)/(\rho^2 + 2\delta_0 j_0^2(t)\sin^2\theta)$. Therefore,

$$T_1(t) \to \frac{j_0^2(t)}{2\sqrt{\pi}} \delta_0 \int_0^{\pi/2} \cos 2\theta \sin\theta d\theta \int_0^{\infty} \frac{\rho^2 \exp(-\rho^2/4)}{\rho^2 + 2\delta_0 j_0^2(t)\sin^2\theta} d\rho. \quad (10)$$

Obviously, the temperature distribution in the conductor depends on how the current density $j_0(t)$ varies at infinity. Let us consider the characteristic modes. Suppose that $j_0(t) \to 0$ as $t \to \infty$. In this case, from Eqs. (8), (9) it follows that $T_1 \to 0$ as $t \to \infty$. This means that the local hot spots that arise are unstable: with time the temperature is distributed uniformly throughout the entire space of the conductor. Under these conditions stable thermal structures cannot exist. Suppose now that $j_0(t) \to \infty$ as $t \to \infty$. In the Appendix we show that (10) leads to the asymptotic form

$$T_1(t) \sim [\ln(8\delta_0 j_0^2(t)) - 6 + C]/4, \quad (11)$$

where $C = 0.57772...$ is Euler's constant. We see that the temperature rises without bound if $j_0(t) \to \infty$.

Between the two modes of $j_0(t)$ variation considered exists a mode of constant current density, $j_0(t) = 1$. In this case the asymptotic form (11) remains valid for $\delta_0 >> 1$, which means that the temperature distribution in the conductor reaches a steady state. To find the distribution of T_1 in the steady state for $j_0(t) = 1$ we rewrite the system of equations (8), (9) as

$$\nabla^2\Phi_1 = -\frac{\partial T_1}{\partial z} - \frac{\partial\Omega}{\partial z}, \quad (12)$$

$$\nabla^2 T_1 = \delta_0\left(2\frac{\partial\Phi_1}{\partial z} + 2T_1 + \Omega\right). \quad (13)$$

The solution of this system of equations for $\delta_0 >> 1$ is the function (for the derivation see Appendix)

$$T_1(r, z) = \frac{1 - 2z^2}{4}\exp(-z^2)\,\mathrm{Ei}(-r^2) - \exp(-r^2 - z^2) +$$

$$+ \frac{1}{4\sqrt{\pi}} \int_0^{\infty} x^2 \exp(-x^2/4)\cos(zx) K_0(x^2\xi/\sqrt{2})\, dx, \quad (14)$$

where $\mathrm{Ei}(-x)$ is an integral exponential function; $K_0(x)$ is a modified Bessel function of the second kind; and $\xi = r/\sqrt{\delta_0}$. For $r = z = 0$ Eq. (14) reduces to (11) with $j_0(t) = 1$.

Figure 1 shows the isotherms T_1 = const for $\delta_0 = 10^4$ and a sketch of the distribution function $T_1(r, z)$. We see that a thermal structure that was asymmetric in the coordinates was formed in the mode of constant current density under consideration. The position of its maxima ($r \approx \pm 1.3$, $z = 0$) and minima ($r = 0$, $z \approx \pm 1.3$) coincide with those determined for the initial stage of heating in [5], if we assume that in our case the inhomogeneity at the initial time was a sphere of radius ≈ 1.3. The condition for the formation of a thermal structure in the problem under consideration is that the variation of the current density with time be a monotonically decreasing function. A general analysis of Eqs. (8), (9) suggests that a structure resembling that shown in Fig. 1 will also exist as $j_0(t) \to \infty$, but it will increase with time.

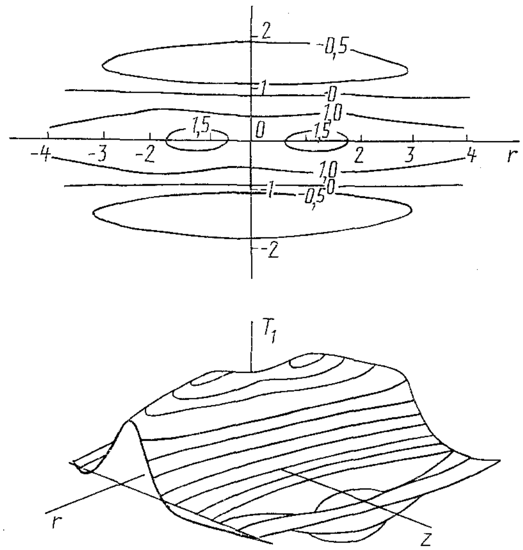

Fig. 1. Sketch of the thermal structure and isotherms in the neighborhood of an inhomogeneity.

In practice which heating mode of the conductor appears will be determined by the ratio between the time constant τ of energy input (time constant of the external electric circuit) and the characteristic time of temperature variation in the conductor. From (7) the latter is of the order of $\sim 1/\delta_0$. If $\tau >> 1/\delta_0$, we can assume that the electric current in the conductor does not vary. We are not particularly considering a mode of oscillatory variation of current. It is clear from analysis that qualitatively it reduces to one of the modes considered above, depending on the relation between the oscillation period and $1/\delta_0$.

As a result of the exponential factor in the expression for the temperature T from (7) at long times even small perturbations of the electrical conductivity $|a| << 1$ give appreciable differences of temperatures at inhomogeneity sites and at infinity. The technological defects, impurities, and other microscopic inhomogeneities that are always present in a real conduction, therefore, become centers of intensive Joule heating; in the final account this can cause uneven heat-induced fracture of the metal.

These fracture effects are important in the design of high-power modern electrophysical devices. This includes devices of pulsed heating of plasmas, generators of strong and ultrastrong magnetic fields, high-current pulsed commutators, etc. [6]. Rapid heating of metals by an electrical current opens up extensive possibilities for obtaining experimental data in the high-temperature range [7]. It was not our aim here to study the mechanism of an electric explosion, but information about the effect of inhomogeneities on thermal processes in conductors can be an important prerequisite for constructing models of the fracture of materials in strong electromagnetic fields.

APPENDIX

1. Expression (10) can be written as

$$
\begin{aligned}
T_1 = & -\frac{1}{2\sqrt{\pi}}\int_0^\infty \rho^2 \exp(-\rho^2/4)\,d\rho \int_0^1 dx + \\
& + \frac{1}{2\sqrt{\pi}}\int_0^\infty \rho^2(\rho^2+D_0)\exp(-\rho^2/4)\,d\rho \int_0^1 \frac{dx}{(\rho^2+2D_0)-2D_0x^2} = \\
& = -1 - \frac{(2D_0)^{3/2}}{16\sqrt{\pi}}\int_0^\infty \sqrt{\frac{y}{1+y}}(2y+1)\times \\
& \times \exp\left(-\frac{D_0 y}{2}\right)\ln\frac{\sqrt{1+y}-1}{\sqrt{1+y}+1}\,dy,
\end{aligned}
\qquad (A1)
$$

where $D_0 = \delta_0 j_0^2(t)$; $y = \rho^2/(2D_0)$. An estimate of the last integral [8]

$$\left|\int_\varepsilon^\infty \sqrt{\frac{y}{1+y}}(2y+1)\exp\left(-\frac{D_0 y}{2}\right)\ln\frac{\sqrt{1+y}-1}{\sqrt{1+y}+1}\,dy\right| \leqslant C\exp\left(-\frac{D_0\varepsilon}{2}\right)$$

makes it possible to write it as a sum $\int_0^\infty = \int_0^\varepsilon + \int_\varepsilon^\infty$ for any $\varepsilon > 0$ and $C = \text{const}$ and to discard the second exponentially small integral on the right for $D_0 >> 1$. Setting $\varepsilon < 1$, expanding the function $\ln[(\sqrt{1+y} - 1)/(\sqrt{1+y} + 1)]$ in a power series

$$[1 + 1{,}5y + O(y^2)]\ln(y/4) - y/2 + O(y^2),$$

integrating (A1) between the limits 0 and ε, and discarding terms of the order of $\ln D_0/D_0$, we obtain (11).

2. To find the solution of the system of equations (12)-(13) we write functions Φ_1 and T_1 as

$$\Phi_1(r,z) = \Phi_{10}(r,z) + Q_0(\xi,z) + \frac{1}{\delta_0}\Phi_{11}(r,z) + \frac{1}{\delta_0}Q_1(\xi,z) + O\left(\frac{1}{\delta_0^2}\right), \tag{A2}$$

$$T_1(r,z) = T_{10}(r,z) + \Pi_0(\xi,z) + \frac{1}{\delta_0}T_{11}(r,z) + \frac{1}{\delta_0}\Pi_1(\xi,z) + O\left(\frac{1}{\delta_0^2}\right), \tag{A3}$$

where $\xi = r/\sqrt{\delta_0}$. Substituting these series into (12)-(13) and equating the terms with the same powers of δ_0, we obtain the equations

$$\nabla^2\Phi_{10} + \nabla_z^2 Q_0 = -(\nabla_z T_{10} + \nabla_z\Pi_0 + \nabla_z\Omega), \tag{A4}$$

$$\nabla_z\Phi_{10} + \nabla_z Q_0 + T_{10} + \Pi_0 + \Omega/2 = 0, \tag{A5}$$

$$\nabla^2\Phi_{11} + \nabla_z^2 Q_1 + \nabla_\xi^2 Q_0 = -(\nabla_z T_{11} + \nabla_z\Pi_1), \tag{A6}$$

$$\nabla^2 T_{10} + \nabla_z^2\Pi_0 = 2\nabla_z\Phi_{11} + 2\nabla_z Q_1 + 2T_{11} + 2\Pi_1, \tag{A7}$$

where for brevity we have denoted the operators $\nabla_z = \partial/\partial z$, $\nabla_z^2 = \partial^2/\partial z^2$,

$$\nabla_\xi^2 = \frac{1}{\xi}\frac{\partial}{\partial\xi}\left(\xi\frac{\partial}{\partial\xi}\right), \quad \nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{\partial^2}{\partial z^2}.$$

Since Q_0, Q_1, Π_0, and Π_1 do not depend on r and Φ_{10}, Φ_{11}, T_{10}, and T_{11} do not depend on ξ, Eqs. (A4)-(A7) split,

$$\nabla^2\Phi_{10} = -\nabla_z T_{10} - \nabla_z\Omega, \tag{A8}$$

$$\nabla_z \Phi_{10} + T_{10} = -\Omega/2, \quad \text{(A9)}$$

$$\nabla_z^2 Q_0 = -\nabla_z \Pi_0, \quad \text{(A10)}$$

$$\nabla_z Q_0 + \Pi_0 = 0, \quad \text{(A11)}$$

$$\nabla_z^2 Q_1 + \nabla_z \Pi_1 + \nabla_\xi^2 Q_0 = 0, \quad \text{(A12)}$$

$$\nabla_z^2 \Pi_0 = 2\nabla_z Q_1 + 2\Pi_1 \quad \text{(A13)}$$

and two more equations for determining Φ_{11} and T_{11}. From (A8)-(A9) we have

$$\Phi_{10} = -\frac{z}{4}\exp(-z^2)\,\mathrm{Ei}(-r^2) + f_{01}(z)\ln(r) + f_{02}(z), \quad \text{(A14)}$$

$$T_{10} = \frac{1-2z^2}{4}\exp(-z^2)\,\mathrm{Ei}(-r^2) - \frac{\exp(-r^2-z^2)}{2} - \frac{df_{01}}{dz}\ln r - \frac{df_{02}}{dz}, \quad \text{(A15)}$$

where $f_{01}(z)$ and $f_{02}(z)$ are as yet undetermined functions of z. Equations (A10)-(A11) degenerate into one equation,

$$\nabla_z Q_0 + \Pi_0 = 0. \quad \text{(A16)}$$

For a unique determination of Q_0 and Π_0 we substitute this equation into the system (A12)-(A13), which can now have a solution if

$$\nabla_\xi^2 Q_0 + \frac{1}{2}\nabla_z^3 \Pi_0 = 0. \quad \text{(A17)}$$

On condition that the solution is finite at infinity, from the system (A16)-(A17) we determine

$$Q_0 = \frac{1}{2\sqrt{\pi}}\int_{-\infty}^{\infty}\exp(-ixz)\,q(x)\,K_0(x^2\xi/\sqrt{2})\,dx, \quad \text{(A18)}$$

$$\Pi_0 = \frac{i}{2\sqrt{\pi}}\int_{-\infty}^{\infty}\exp(-ixz)\,xq(x)\,K_0(x^2\xi/\sqrt{2})\,dx, \quad \text{(A19)}$$

where q(x) is as yet an undetermined function.

We substitute solutions (A14), (A15), (A18), and (A19) into (A2)-(A3), discard the terms proportional to $1/\delta_0$, and from the condition that Φ_1 and T_1 be finite at the center and vanish at infinity we find the functions f_{01}, f_{02}, and q(x). As a result, we obtain (14) for T_1 and a similar expression for Φ_1.

NOTATION

Dimensionless quantities: r, z, cylindrical coordinates; t, time; j, current density; σ, electrical conductivity; T, temperature; a, inhomogeneity parameter; Ω, function of the geometric shape of the inhomogeneity; E, electric field strength; Φ, electric potential; δ_0, system parameter; $j_0(t)$, electric-current density at infinity; T_1, function describing the temperature distribution in the conductor (7). Dimensional quantities: α, temperature resistance coefficient; σ_0, initial electrical conductivity of the conductor at infinity; χ, thermal conductivity; γ, density; c, heat capacity; R characteristic inhomogeneity dimension; J_0, amplitude of the current density.

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THERMODYNAMIC EQUILIBRIUM OF A GAS MIXTURE WITH A SOLUTION IN THE CONDENSED PHASE

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A thermodynamic analysis is made of the equilibrium of a gas mixture (using a nitrogen–hydrogen mixture as an example) with a solid solution of its components. The processes occurring in the surface layer and interphase exchange are taken into account. The equilibrium characteristics are determined in terms of the equilibrium constants of the intermediate stages of the surface reactions.

The penetration of a gas into a condensed material and the dissolution of gas-mixture components in the solid phase is a complex process consisting of a series of elementary stages: adsorption, desorption, dissociation, and recombination. The kinetics of such a process, the concentration of the dissolved gases, and other characteristics of the interaction of the gas with the solid depend on the rates of the surface reactions and the elementary stages [1]. Under certain conditions these stages can have a decisive effect on the process rate. Traditional methods of describing equilibrium solubility based on directly equating the chemical potentials of the dissolved atoms in the various phases and on the surface are often inapplicable. This is because, e.g., atoms can appear on the surface as a result of dissociation, catalyzed by the solid phase, and may be absent in the gas phase, which contains only molecular gas. Below we make a thermodynamic analysis of such a system with allowance for the dissolution of the gas-mixture components, which is of interest for the chemical and thermal treatment of materials.

Specifically, we consider a nitrogen–hydrogen mixture that contains nitrogen, hydrogen, and ammonia and interacts with solutions of the components in the solid phase, taking into account the effect of the surface of the solid catalyst. This thermodynamic system contains two bulk phases (gas and solid) and one surface phase. We note that heterogeneous processes

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